**STAT 6106: Applied Bayesian Methods**

### Mid-term Examination

### Oct. 13th, 2017, 8:00pm-9:30pm (90min)

**Policies**:

•This is an open-book exam. You may use any printed/written materials and a calculator.

•All work must be your own. You must not give or receive aid of any kind.

•This handout includes 1-page questions (3 problems).

•Only your writing on the answer book will be graded.

**Problem 1 (10 points).** What is the relationship between the uncertainty in the prior and the uncertainty in the posterior?

**Problem 2 (50 points).** Assume the weight of CUHK students follows a normal distribution with unknown mean and known variance 400. Suppose we got an i.i.d. sample of *10* CUHK students’ weight, where the average weight is 150.

**(a) (15 points)** Suppose we know the mean weight of HKU students is around 180 and the typical difference of the mean student weight across the universities in Hong Kong is around 40. Please perform Bayesian inference for the mean weight of CUHK students.

**(b). (20 points)** Suppose we are going to sample a new CUHK student’s weight randomly. What is your prediction of this new weight?

**(c). (15 points)** Give a 95% posterior interval for the mean weight of CUHK students and a 95% interval for the new weight. Which of the two variables has big uncertainty?

**Problem 3 (40 points).** There is a special coin. The probability to get its Head side is equal to ***p***. We ask 10 students to each independently toss the coin until he/she gets the Head side. Denote the number of trials to get the Head side as ***X***. We know that ***X*** follows a First Success distribution with probability mass function , with and .

The data observed ***X*** from these 10 students are: 2,1,3,1,4,4,16,3,5,1.

**(a) (10 points)** What type of distribution is the conjugate prior distribution for ***p***? Give the distribution name and its general density function.

**(b). (5 points)** We actually do not have any prior information about ***p***. How to choose a specific distribution as your prior from the conjugate distribution family in **(a)**?

**(c). (10 points)** Please derive the posterior distribution using the observed data from the 10 students.

**(d). (15 points)** Now suppose you are going to toss this coin until you get the Head side. How many trials will you need to get the Head side?